Classical Derivation of Planck’s Relation and Constant through the Poynting Vector

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DERIVATION OF THE PLANCK RELATION

Planck’s relation $E = h\nu$ establishes a direct proportionality between the energy of a photon and the frequency of the electromagnetic wave. In the conventional framework of quantum mechanics, Planck’s constant is introduced as a fundamental constant [1]. Its value is determined experimentally and accepted as a starting point. In this work, however, it is shown that the relation $E = h\nu$, and thus Planck’s constant h, can be derived using classical electromagnetic theory. A similar approach can be used in the generalized electrogravitational case [2]. In $Fig. 1$, it can be seen how each electromagnetic wave with periodicity 2π has an associated gravitational wave of periodicity π (electrogravitational wave). Here, **E** and **B** are the electric and magnetic fields, while **g** and **K** are the gravitational and cogravitational fields [2].

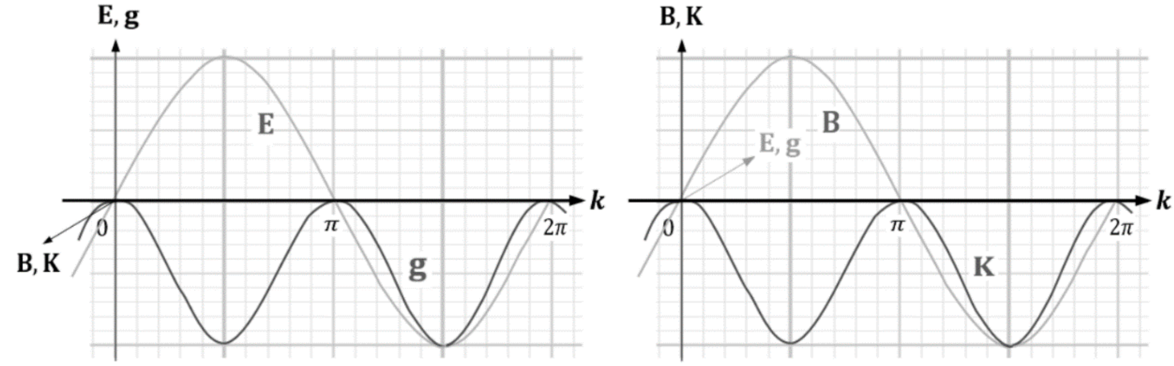

Fig.1

To proceed, we consider particles as point-like, isotropic emitters of electromagnetic waves, and therefore of photons. It is assumed that the radiated power is uniformly distributed over a spherical wavefront with surface area $A = 4\pi r^2$, where r represents the distance from the point of emission. We want to determine the amount of energy radiated by the source in the case of monochromatic emission. In the electrogravitational [2] case, the gravitational counterparts are used. This also applies to the emitted particles, which are now graviphotons [2]. Given a generic charge q_i, it can be expressed as a multiple of the elementary charge $q_i = z_i \cdot q_e$ $(z_i \in \mathbb{Z})$. We can substitute this relationship into Maxwell's equations, so that they depend on the number density of charges ρ_z. Among the solutions of Maxwell's equations are electromagnetic waves. These describe the behavior of the electric **E** and magnetic **B** fields, which can be expressed by their root mean square values over the period:

$$\mathbf{E}_{RMS} = z\frac{Kq_e}{r^2}\,\hat{\mathbf{k}}_\mathbf{1} \qquad \mathbf{B}_{RMS} = z\frac{Kq_e}{c\,r^2}\,\hat{\mathbf{k}}_\mathbf{2} \tag{1.1}$$

Here $\hat{\mathbf{k}}_\mathbf{1}$ and $\hat{\mathbf{k}}_\mathbf{2}$ represent the unit vectors associated with the fields **E** and **B**, respectively. Given the Electromagnetic Poynting vector **S**, defined by the relation:

$$\mathbf{S} = \frac{c^2}{4\pi K}\,\mathbf{E}\times\mathbf{B} \tag{1.2}$$

To determine the power radiated by a point source during the time interval $t = r/c$, in the case of monochromatic emission, it is necessary to integrate over the spherical surface area $4\pi r^2$ of the wavefront. Then, since monochromatic emission implies a collective motion of z charges, one must multiply by the number of oscillations n during the interval t (assuming that one photon is emitted for each oscillation of a single charge, otherwise another multiplicative constant would need to be introduced in the calculations). Since $r^2 = c^2t^2$, the power radiated across the entire surface is:

$$P = \int_A \mathbf{S}\cdot\hat{\boldsymbol{n}}\,dA = (4\pi r^2)\frac{c^2}{4\pi K}\left(z\frac{Kq_e}{r^2}n\cdot z\frac{Kq_e}{c\,r^2}n\right) = \frac{Kq_e^2}{c}\frac{z^2n^2}{t^2} \tag{1.3}$$

Given that $n/t = v$ represents the frequency, if we multiply by $2\pi/zv$ (since 2π represents the integral over a full cycle), and then by the inverse of the fine-structure constant $1/\alpha \approx 137$ (interpreted here as an empirical parameter acting as a coupling coefficient between light and matter, since α can be determined through experiments that do not require knowledge of h [3]), we obtain Planck's relation for the energy:

$$E = |z|\frac{2\pi Kq_e^2}{c\alpha}v = |z|\frac{Z_0q_e^2}{2\alpha}v = |z|hv \tag{1.4}$$

Here, $h \approx 6.626\cdot 10^{-34}J\cdot s$ represents the Planck constant, Z_0 is the impedance of free space, K is the Coulomb constant, $q_e \approx 1.6\cdot 10^{-19}C$ is the elementary electric charge, and $|z|$ is the number of emitters. If $|z| = 1$, the relation reduces to $E = hv$.

BIBLIOGRAPHY

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